Reply to "Electrodynamic force law controversy"

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Our paper [Phys. Rev. E **58**, 2505 (1998)] confirmed the validity of both Ampère and Grassmann's force law even for the action exerted on a part of a current loop. Since that part can be an element of a circuit, both force laws also predict the same internal stresses and the same recoil for a railgun. Graneau and Graneau [preceding paper, Phys. Rev. E **63**, 058601 (2001)] neglected the action on the breech of the railgun, an action that produces the same recoil for both force laws. The reaction to the force exerted on the armature does not act on the rails but on the breech that, simply because of symmetry, undergoes a force equal and opposite to the one acting on the armature.

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We agree with Graneau and Graneau [1] that our experiment [2] proves the validity of both the Ampère and Lorentz force law when applied to closed circuits. In fact, the main motivation of our experiment was to repeat, in a better way, the experiments of Pappas [3] and Phipps and Phipps [4] who claimed to have disproved the Lorentz force law. However, in the Introduction of our paper [2], we pointed out that Laplace's first law

$$d\mathbf{B}=(\mu_0/4\pi)I_1 d\mathbf{s}_1\times(\mathbf{r}-\mathbf{r}')|\mathbf{r}-\mathbf{r}'|^{-3} \quad (1)$$

is an approximation to the Lienard-Wiechert law for $v/c \to 0$ and for negligible acceleration $\mathbf{a}$. Now, the Lienard-Wiechert expression is the solution to the Maxwell equations for a pointlike electric charge q, and there is nothing in physics better proved than the circuital laws that the Maxwell equations are derived from. Laplace's second law

$$\delta\mathbf{F}_2=I_2\delta\mathbf{s}_2\times\mathbf{B} \quad (2)$$

has also been checked very well. Substituting Eq. (1) into Eq. (2), we obtain the Grassmann (also called Biot-Savart) force law between current elements

$$\delta\mathbf{F}_2=\frac{\mu_0}{4\pi}I_1 I_2\delta\mathbf{s}_2\times\left(d\mathbf{s}_1\times\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\right), \quad (3)$$

which is therefore a consequence of the Maxwell equations and the experimental expression (2) of the force on a current element. On the contrary, Ampère's law is not even an approximation of the solutions of the Maxwell equations.

Graneau and Graneau [1] object that one needs to perform measurements of the internal reaction force distribution in an isolated current loop to find where the conflict occurs. But internal reactions are defined as the stresses one must apply to the cross section of an element out from the body (the circuit in our case) to keep it in equilibrium. Now, this is just what has been performed to experimentally prove Eq. (2), by mechanically, but not electrically, isolating any small section of a circuit.

Implicitly in Ref. [1] and explicitly in Ref. [5], the two Graneaus state that experiments favor the Lorentz force

$$\delta\mathbf{F}=\delta q\mathbf{v}\times\mathbf{B} \quad (4)$$

in electron guns (i.e., on free electrons) and Ampère's law on current elements. In fact, Eq. (2) can be derived from Eq. (4) that is experimentally proved up to nine significant figures in mass spectrometers. The conduction current in a wire element is due to electrons moving at a speed $v=400$ km/s and scattering against the ion lattice. The acceleration due to a small electric field $\mathbf{E}$ inside the wire produces an almost imperceptible bending of the electron trajectory along their free flights between two subsequent scatterings. The average speed of n electrons is

$$\langle\mathbf{v}\rangle=\frac{1}{n}\sum_{i=1}^{n}\mathbf{v}_i \quad (5)$$

and, in a wire in which a high current density is flowing, $\langle\mathbf{v}\rangle$ is of the order of 0.01 cm/s. The resultant force on a wire element of length δs and cross section S immersed in a magnetic field $\mathbf{B}$ is, according to Eqs. (4) and (5),

$$\delta\mathbf{F}=\sum_{i=1}^{n}e\mathbf{v}_i\times\mathbf{B}=en\langle\mathbf{v}\rangle\times\mathbf{B}, \quad (6)$$

where e is the electron charge. Now it is $n=N\,\mathbf{S}\cdot\delta\mathbf{s}$, where N is the numerical concentration and $\mathbf{S}$ the oriented, vector cross section, so that, being $\hat{\delta\mathbf{s}}=\delta\mathbf{s}/\delta s=\langle\hat{\mathbf{v}}\rangle=\langle\mathbf{v}\rangle/|\mathbf{v}|$, we can write

$$e\,n\langle\mathbf{v}\rangle=e\,N\,\mathbf{S}\cdot\delta\mathbf{s}\langle\mathbf{v}\rangle=e\,N\langle\mathbf{v}\rangle\cdot\mathbf{S}\,\delta\mathbf{s}=\mathbf{j}\cdot\mathbf{S}\,\delta\mathbf{s}=I\,\delta\mathbf{s}, \quad (7)$$

where $\mathbf{j}=e\,N\langle\mathbf{v}\rangle=\rho\langle\mathbf{v}\rangle$ is the current density and $\rho=e\,N$ the charge density of the electrons. Substituting Eq. (7) into Eq. (6) we obtain Eq. (2), i.e., Laplace's second law that is therefore equivalent, or derivable, from the Lorentz law, Eq. (4).

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Consequently, it is not possible to state that experiments favor Eq. (4) for free electrons and Ampère's law for current elements.

The main subject of the present Graneau comment [1] is the recoil forces on a railgun. They start by stating that the Ampère electrodynamics predicts that the rails are pushed back longitudinally by the armature toward the breech of the railgun. Then, quoting Feynman's lecture on physics but adding some ununderstandable statement: ''the recoil force corresponding to the armature acceleration must therefore cause the deceleration of the incoming energy,'' they correctly state that ''the electrodynamic forces are perpendicular to the current in the conductor with zero longitudinal component.'' But the force on one of the rails is just the force on a part of a circuit due to the whole circuit. This was just what we did in our experiment [2] and we [6] have proved, and the Graneaus recognize, that both Grassmann and Ampère laws give the same results. Why do the two Graneaus now contradict themselves by stating that Ampère's law predicts a force on one of the rails different from Grassmann's? The recoil acts, for both Grassmann and Ampère force laws, on the breech of the railgun. Consider a symmetric railgun as in Fig. 1. Because of symmetry, the force on the armature is equal and opposite to that on the breech. We have shown the breech in Fig. 1 with the same thick line as the armature, differently from the figure of Ref. [1], where the thin line denoting the breech perhaps invites one to neglect the force on it.

We conclude that, for closed circuits, there is complete agreements for both Ampère and Grassmann force laws, even concerning recoils and internal stresses. However, there are not only experiments on closed circuits at low frequencies for the current flowing in them. There are also very accurate experiments on accelerated electrons, as those in modern synchrotrons. These experiments show that the Lorentz force (equivalent to Laplace's second law) is always valid but the field produced by accelerated charges is given by the complete solutions of Lienard and Wiechert. Consequently, Laplace's first law is no longer valid in these cases since it is an approximation of the Lienard-Wiechert solution not only for $v \ll c$ but also because it neglects the acceleration (or radiation) term. It is the latter term that produces the radiation of electromagnetic waves (as the light from a lamp) and, in the case of synchrotrons, the recently well-studied synchrotron light. Now Ampère's electrodynamics gives no hints as to how to face radiation and relativistic corrections.

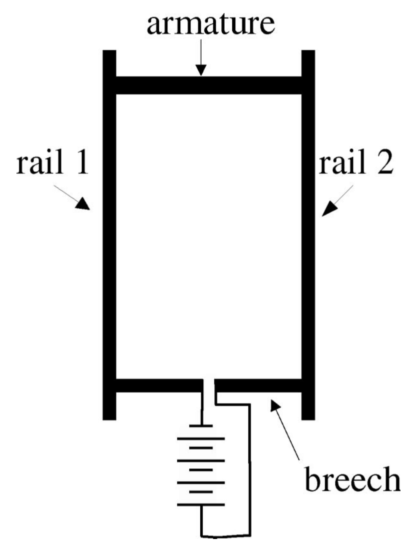

FIG. 1. Sketch of the railgun, where the armature and the breech are shown in a symmetrical way, thus pointing out that the reaction force on the breech is equal and opposite that on the armature.

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